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Inherently Incomplete Finite Element Model and Its Effects on Model Updating

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Some of the questions regarding the use of measured data to improve a finite element model of a structure to make it better able to predict the dynamic behavior of the structure are addressed. The issues discussed include the following: All such models are truly reduced models, the parameters of a reduced model of a linear structure vary with frequency, a linear model of a linear structure has a limited frequency range of applicability, and there are an infinite number of reasonable models that can match measured data. These and other related concepts are discussed and demonstrated. Of course, because there is no unique solution, questions arise as to how to select the best one, and are there different best models for different applications. It is concluded that new paths of research in this important field are necessary.

Nomenclature

$\{f\}$	=	vector of applied forces
$[K]$	=	stiffness matrix
$[K_R]$	=	reduced-order stiffness matrix
$[M]$	=	mass matrix
$[M_R]$	=	reduced-order mass matrix
$[m]$	=	generalized mass matrix
m_i	=	generalized mass of mode i
$\{x\}$	=	vector of displacements
$[\Phi]$	=	eigenvector matrix
$[\Omega^2]$	=	diagonal matrix of natural frequencies squared
ω	=	frequency
ω_i	=	natural frequency of mode i

Introduction

FINITE element modeling is of great importance in many branches of engineering. This highly developed technology has allowed the detailed analysis of complex structures and has resulted in significant advances in the ability to design systems for a large variety of purposes.

However, a finite element model (FEM) is an approximate discrete analytical model of a continuous structure. If one wished to develop a dynamic FEM model of such a structure, one approach would be to select the degrees of freedom (DOF) so as to represent the structural characteristics in sufficient detail to satisfy the purpose of the analysis. The simplest form of the model may consist of a lumped mass at each DOF with linear springs connecting the adjacent masses. This would result in a banded stiffness matrix representing the load paths connecting the lumped masses along with a diagonal mass matrix.

One might also model the same structure with more DOF than just described and then analytically reduce them to the same DOF as selected earlier. In this case the resulting stiffness matrix would be at least partially filled, representing actual load paths that were omitted using this procedure.

If the structure were truly linear, the reduced stiffness matrix would also be linear. Even if the simple lumped mass scheme is used, however, the reduced mass matrix, which is now no longer a diagonal matrix, would vary with frequency.[1−3]

Which of the two models is more representative of the actual infinite DOF structure? It takes little logical reasoning to conclude that the second one is the better representation. Why, then, do many practioners in the field of improving the model (stiffness matrix alone, in many cases) based on test data, insist that an improved analytical model of a structure maintain the same banded formulation as the original simplified FEM? One may conclude that we are so FEM oriented that we see little, if any, significant difference between the FEM and the actual structure.

It is apparent, but rarely acknowledged, that for any given structure, there are many acceptable models. None of these models, however, is a perfect representation of the structure.

Since about 1970, there has been a continuous stream of publications addressing the problem of improving analytical dynamic models through the use of test data. The well-known survey by Mottershead and Friswell[4] lists well over 200 such publications between 1970 and 1993. This interesting and exciting work is still continuing, as illustrated by recent innovative examples by Chiang and Huang[5] and Abdula et al.[6] Although it is often mentioned that the solution is not unique, it is common for this consideration to be completely ignored after its initial acknowledgment.

Some of the published procedures that are concentrated on maintaining the form of the banded FEM stiffness matrix while improving the agreement with test data represent very fine and interesting work (for example, Refs. 7 and 8). Some, such as Ref. 8, apply the method to structures such as trusses, where the banded concept may have somewhat more credibility.

It is well known and recognized by most practioners that the FEM is a good representation of the actual structure only over a very limited lower-frequency range. At best, only a relatively small set of analytical natural frequencies and normal modes correspond to those of the actual structure. It is also true that an FEM that predicted all, or many, of the modes and frequencies would be a very poor representation of the physical structural properties. Even so, it is common to talk about full sets of data (meaning N modes and frequencies of an N-DOF model) and believe that there is a true (or unique) improved model that will exactly match all of the test data.

In the following, it will be discussed and demonstrated that a good FEM, including an updated version, must be incomplete in that it cannot represent all of the characteristics of the structure. It follows that there can be many improved models that will predict the test results (including the lower-order modes and natural frequencies) and be close to the original FEM.[1,9]

A major question remains unanswered. Because there are many good models of any structure, how does one select a particular model for a particular application? This question opens a field for study that has been rarely addressed in the past.

This presentation is limited to a simple structure and model and simple data; that is, linear structure, linear model, and exact measured modes and frequencies. However, it must be obvious that the main conclusion obtained applies equally well to many other procedures that use more realistic measured data to improve more

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complex analytical models. This conclusion is that there can be no unique corrected dynamic model of structure as long as the model has fewer DOF than the actual structure.

Conditions of the Discussion

The discussion of this topic will be limited to the following conditions.

The true actual structure has an infinite number of DOF. In this study, the simulated actual structure is represented by an FEM that has more DOF than the model and is linear and undamped.

The simulated test data consist of normal modes and natural frequencies of the simulated structure. It is assumed that they are without error. Whereas the simulated measured modes and frequencies are known, the true values of the mass and stiffness of the actual structure are unknown. However, estimated values are available. The dynamic equations representing steady-state responses will be employed.

The simulated structure is represented by point masses and weightless springs. The masses are attached to all adjacent masses (through the springs), thus, they represent a realistic structure such as a beam with finite dimensions or an aircraft fuselage. All masses and stiffnesses are treated as being without specific dimensions but are assumed to be consistent.

It is believed that the limitations just described do not limit the conclusions that there is no unique FEM representing a realistic structure.

Reduced Models

Any FEM of a structure or any data obtained from discrete test measurements involve a model with fewer DOF than the actual structure.

The basic equation representing the steady-state response of the structure is

$$([K] - \omega^2[M])\{x\} = \{f\} \tag{1}$$

If the equation is partitioned so that the subscript t represents the DOF of the model (and the test) and u represents the untested DOF, then

$$\left(\begin{bmatrix} K_1 & K_2 \\ K_2^T & K_4 \end{bmatrix} - \omega^2 \begin{bmatrix} M_1 & M_2 \\ M_2^T & M_4 \end{bmatrix}\right)\begin{Bmatrix} x_t \\ x_u \end{Bmatrix} = \begin{Bmatrix} f_t \\ f_u \end{Bmatrix} \tag{2}$$

Assuming that no forces are applied except at the tested DOF, that is, $f_u = 0$, this can be reduced[2] to

$$\big([K_R] - \omega^2[M_R]\big)\{x_t\} = \{f_t\} \tag{3}$$

where

$$[K_R] = [K_1] - [K_2][K_4]^{-1}[K_2]^T \tag{4}$$

and (for convenience, assuming $[M_2] = 0$)

$$[M_R] = [M_1] + [K_2][K_4]^{-1}[M_4]\big(I - \omega^2[K_4]^{-1}[M_4]\big)^{-1} \times [K_4]^{-1}[K_2]^T \tag{5}$$

These equations are exact. However, note that the equations of a linear system, represented by a reduced number of DOF contain a mass matrix that is a function of the frequency of excitation.

Illustrative Example

As an illustration of the consequences of all FEM being reduced models, consider the following cantilevered beam-like structure[1] shown in Fig. 1. The DOF are numbered 1–12 and represent point masses connected by linear weightless springs. For the purposes of discussion, this structure shall be considered the true structure, which will be reduced to a simpler model. The model, a six-DOF representation, is shown in Fig. 2.

Table 1 Element masses

Element	Mass
1	1
2	1
3	2
4	2
5	0.5
6	0.2
7	0.5
8	0.5
9	0.5
10	0.5
11	0.4
12	0.3

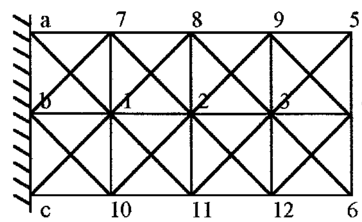

Fig. 1 Simple structure DOF.

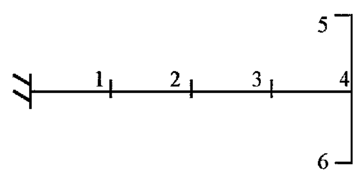

Fig. 2 Model of simple structure.

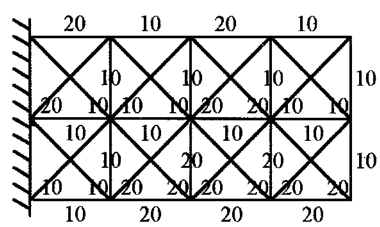

Fig. 3 Stiffness of elements.

True Structure

In the true structure (Fig. 1), all of the load paths are assumed to be linear weightless springs with stiffness as shown in Fig. 3. The stiffnesses were assigned so as to avoid symmetry in the structure, which could confuse the issues. The assumed masses of the elements of the structure are given in Table 1.

The stiffness matrices [as in Eq. (2)] are then

$$[K_1] = \begin{bmatrix} 90 & -10 & 0 & 0 & 0 & 0 \\ -10 & 120 & -10 & 0 & 0 & 0 \\ 0 & -10 & 120 & -10 & -10 & -20 \\ 0 & 0 & -10 & 60 & -10 & -10 \\ 0 & 0 & -10 & -10 & 30 & 0 \\ 0 & 0 & -20 & -10 & 0 & 50 \end{bmatrix}$$

$$[K_2] = \begin{bmatrix} -10 & -10 & 0 & -10 & -20 & 0 \\ -10 & -10 & -20 & -20 & -20 & -20 \\ 0 & -20 & -10 & 0 & -20 & -20 \\ 0 & 0 & -10 & 0 & 0 & -20 \\ 0 & 0 & -10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -20 \end{bmatrix}$$

$$[K_4] = \begin{bmatrix} 70 & -10 & 0 & 0 & 0 & 0 \\ -10 & 70 & -20 & 0 & 0 & 0 \\ 0 & -20 & 70 & 0 & 0 & 0 \\ 0 & 0 & 0 & 70 & -20 & 0 \\ 0 & 0 & 0 & -20 & 100 & -20 \\ 0 & 0 & 0 & 0 & -20 & 100 \end{bmatrix}$$

Table 2 Modes and frequencies of structure

Mode	Frequency ω, rad/s											
	2.09	5.90	7.62	8.16	10.03	10.43	11.99	12.70	13.79	14.63	16.36	20.09
1	0.386	−0.565	−0.310	0.594	−0.900	−0.500	−0.164	−0.179	0.065	−0.028	−0.094	0.021
2	0.655	−0.590	−0.192	0.319	0.573	0.512	0.106	−0.498	−0.224	−0.168	−0.120	−0.053
3	0.850	−0.475	0.141	−0.691	−0.224	−0.038	0.015	0.019	0.081	−0.112	−0.019	0
4	1.0	1.0	−0.181	0.022	−0.014	−0.039	0.006	0.029	−0.013	−0.055	0.001	−0.018
5	0.956	0.168	1.0	1.0	−0.376	0.219	0.108	−0.004	−0.164	0.007	−0.02	0
6	0.882	−0.057	−0.031	−0.465	−0.165	0.203	0.079	−0.241	−0.155	1.0	−0.729	−0.646
7	0.254	−0.331	−0.128	0.285	0.200	−0.594	1.0	0.173	0.455	0.042	0.030	0.002
8	0.680	−0.853	−0.021	0.134	0.722	−0.980	−0.127	0.492	−0.980	0.042	0.025	0.001
9	0.807	−0.315	0.131	0.337	1.0	−0.457	−0.474	−0.030	1.0	0.111	0.032	0.009
10	0.439	−0.537	−0.263	0.403	−0.102	1.0	−0.061	1.0	0.190	−0.026	−0.261	0.057
11	0.641	−0.538	−0.191	0.123	−0.224	0.480	−0.018	0.054	−0.046	0.230	1.0	−0.335
12	0.816	−0.147	−0.110	−0.173	−0.016	0.344	0.066	−0.247	−0.167	0.500	0.143	1.0

The normal modes and natural frequencies of the structure are shown in Table 2.

Model

The parameters of the model as given by Eqs. (4) and (5) (for $\omega = 0$) are as follows:

$$[K_R] = \begin{bmatrix} 79.3 & -24.9 & -10.2 & -1.5 & -0.5 & -1.0 \\ -24.9 & 88.7 & -31.0 & -9.0 & -3.6 & -5.3 \\ -10.2 & -31.0 & 99.9 & -17.5 & -12.5 & -25.1 \\ -1.5 & -9.0 & -17.5 & 54.3 & -11.6 & -14.2 \\ -0.5 & -3.6 & -12.5 & -11.6 & 28.4 & 0 \\ -1.0 & -5.3 & -25.1 & -14.2 & 0 & 45.8 \end{bmatrix}$$

$$[M_R]_{(\omega=0)} = \begin{bmatrix} 1.08 & 0.13 & 0.08 & 0.02 & 0.01 & 0.01 \\ 0.13 & 1.26 & 0.17 & 0.06 & 0.04 & 0.03 \\ 0.08 & 0.07 & 2.15 & 0.05 & 0.03 & 0.02 \\ 0.02 & 0.06 & 0.05 & 2.03 & 0.01 & 0.01 \\ 0.01 & 0.04 & 0.03 & 0.01 & 0.51 & 0 \\ 0.01 & 0.03 & 0.02 & 0.01 & 0 & 0.21 \end{bmatrix}$$

As a sample indication of the effect of frequency on the mass matrix, the matrix at the natural frequency of mode 4 is given here:

$$[M_R]_{(\omega=8.16)} = \begin{bmatrix} 1.20 & 0.33 & 0.22 & 0.06 & 0.04 & 0.02 \\ 0.33 & 1.67 & 0.47 & 0.15 & 0.11 & 0.04 \\ 0.22 & 0.47 & 2.40 & 0.13 & 0.09 & 0.03 \\ 0.06 & 0.15 & 0.13 & 2.06 & 0.04 & 0.02 \\ 0.04 & 0.11 & 0.09 & 0.04 & 0.54 & 0 \\ 0.02 & 0.04 & 0.03 & 0.02 & 0 & 0.22 \end{bmatrix}$$

An illustration of the effect of the frequency dependent mass matrix is given in Tables 3–5, where modes and frequencies using different mass matrices are shown ($[M_{\text{full}}]$ refers to the full model of the structure as shown in Fig. 1, which can be interpreted as the correct values). Note that $\omega = 8.16$ is the frequency of the correct fourth mode.

Table 3 Mode 1 using different mass matrices

DOF	Mass matrices (frequency ω) $[M_R]_{(\omega=0)}$ (2.097)	$[M_R]_{(\omega=8.16)}$ (1.852)	$[M]_{\text{full}}$ (2.090)
1	0.386	0.413	0.386
2	0.654	0.695	0.655
3	0.849	0.876	0.850
4	1.0	1.0	1.0
5	0.955	0.975	0.955
6	0.881	0.899	0.882

Table 4 Mode 3 using different mass matrices

DOF	Mass matrices (frequency ω) $[M_R]_{(\omega=0)}$ (7.717)	$[M_R]_{(\omega=8.16)}$ (7.602)	$[M]_{\text{full}}$ (7.628)
1	−0.280	−0.316	−0.310
2	−0.181	−0.193	−0.192
3	0.077	0.156	0.141
4	−0.151	−0.190	−0.181
5	1.0	1.0	1.0
6	−0.058	−0.026	−0.031

Table 5 Mode 4 using different mass matrices

DOF	Mass matrices (frequency ω) $[M_R]_{(\omega=0)}$ (8.299)	$[M_R]_{(\omega=8.16)}$ (8.163)	$[M]_{\text{full}}$ (8.163)
1	0.933	0.596	0.594
2	0.489	0.321	0.319
3	−0.783	−0.694	−0.691
4	0.028	0.023	0.022
5	1.0	1.0	1.0
6	−0.493	−0.467	−0.465

Preliminary Interpretation of Results

Note that the true mathematical representation of the simple structure (for example, Fig. 1) consists of a banded stiffness matrix and a diagonal mass matrix. The system is linear. The true mathematical representation of the reduced version (for example, Fig. 2) of this structure consists of a filled stiffness matrix and a filled mass matrix that varies with frequency.

A linearized version of the reduced model (i.e., using a constant mass matrix) cannot match all of the characteristics of the structure.

What is called the model here is considered to be the representation of the structure being analyzed by an engineer, who assumes it to be linear with a banded stiffness matrix and a diagonal mass matrix. The structure is considered to be one level above this. If the engineer were modeling what has been called the structure, one must recognize that that also is truly a reduced model and it, then, would have filled matrices with the same type of nonconstant parameters as in the example.

Load Path Considerations

One row of the stiffness matrix represents the forces that must be applied at each of the DOF to produce a unit displacement at one DOF and zero displacement at all of the others. Nonzero elements represent the load paths of the structure. If a DOF is removed from the model of the structure, new load paths appear.

For example, in Fig. 1, there is no load path between DOF 7 and 9 because the loads are blocked by DOF 8 and 2. In the model (Fig. 2), there is certainly a load path between DOF 1 and 3 because DOF

Table 6 Spring rates to ground of model

DOF	Spring rate to ground
1	41.06
2	14.87
3	3.60
4	0.51
5	0.26
6	0.25

7–12 no longer exist. In establishing the equations of the model (as represented in Fig. 2), it would be unusual for these load paths to be included.

Note that, in formulating the representation in Fig. 1, it was assumed that there was no load path between DOF 1 and 9, for example. Actually, however, because this is a continuous structure, there are certainly connections between all of the DOF of the structure. This would fill all the zero elements in $[K_1]$, $[K_2]$, and $[K_4]$.

An interesting aspect of the load paths is the connection to ground. For a model such as is being discussed, the sum of the elements in a row is the spring rate to ground of the element represented by that row. For example, in the stiffness matrix representing the structure of Fig. 1, only the rows corresponding to elements 1, 7, and 10 add to other than zero. No other element has a direct connection to ground.

Consider now the model in Fig. 2. If there is no consideration that this is a reduced version of Fig. 1, then the only element connected to ground is element 1. This is what one would expect to see if this model were considered to be representative of the structure.

When the stiffness matrix of the model is obtained by considering it to be a reduced model, then something very different results. Note that by removing the designation of elements 7–12 as DOF, one does not remove the connections between these points. Thus, one should expect that all the remaining elements will have load paths to ground. Table 6 shows the values of the load paths to ground from each DOF of the model, obtained by summing the rows of $[K_R]$.

Whereas it is assumed that the person modeling the structure has no detailed knowledge of the higher-level structure from which the model is a reduced version, that person certainly should be aware of such realistic characteristics. Assuming that the stiffness matrix has a simplistic formulation is not acceptable.

This statement should, of course, be qualified. If the meaningful dynamic characteristics of the structure can be represented by the number of normal modes and natural frequencies that represent a small fraction of the order of the model, then such modeling should be considered useful and normal. This is, of course, the way that the very meaningful technology of finite element analysis operates.

Multiple Acceptable Solutions

Based on the preceding discussions, it is apparent that there is no single acceptable linear dynamic model of a given structure. To illustrate this point, it shall be assumed that an acceptable model of the given structure (having six DOF) will predict the first four normal modes and natural frequencies.

It is possible to express the stiffness matrix in terms of the normal modes of the structure, as follows:

$$[K] = [M][\Phi][\Omega^2][m]^{-1}[\Phi]^T[M] \tag{6}$$

This procedure shall be applied to the reduced model of the structure, given earlier, by the use of the mass matrix for $\omega = 0$. The normal modes for this case are shown in Table 7.

If Eq. (6) were applied to the preceding data, the resulting matrix would be exactly the same as $[K_R]$, shown earlier.

If the frequencies of modes 5 and 6 were arbitrarily changed to 12 and 17, one would not expect that the response of the model would significantly change for frequencies below that of the third or forth modes. Such a model would be likely as acceptable as the original. The resulting stiffness matrix, by the use of Eq. (6), is

Table 7 Normal modes of model[a]

	Mode (frequency ω)					
DOF	1 (2.10)	2 (6.01)	3 (7.72)	4 (8.30)	5 (10.41)	6 (15.11)
1	0.386	−0.578	−0.278	0.933	−0.905	0.004
2	0.654	−0.612	−0.181	0.489	1.0	−0.029
3	0.849	−0.551	0.077	−0.783	−0.221	−0.070
4	1.0	1.0	−0.151	0.028	−0.036	−0.037
5	0.955	0.112	1.0	1.0	−0.080	0.025
6	0.881	−0.108	−0.058	−0.494	−0.017	1.0

[a]Mass matrix for $\omega = 0$.

$[K_R](\omega_5, \omega_6 = 12.0, 17.0)$

$$= \begin{bmatrix} 92.6 & -42.5 & -3.7 & -1.0 & -0.4 & -0.9 \\ -42.5 & 110.5 & -37.4 & -9.3 & -4.2 & -6.5 \\ -3.7 & -37.4 & 106.6 & -14.8 & -12.4 & -32.9 \\ -1.0 & -9.3 & -14.8 & 55.5 & -11.7 & -17.9 \\ -0.4 & -4.2 & -12.4 & -11.7 & 28.4 & 0.5 \\ -0.9 & -6.5 & -32.9 & -17.9 & 0.5 & 57.9 \end{bmatrix}$$

This should be compared with the original matrix:

$[K_R](\text{original}: \omega_5, \omega_6 = 10.04, 15.11)$

$$= \begin{bmatrix} 79.3 & -24.9 & -10.2 & -1.5 & -0.5 & -1.0 \\ -24.9 & 88.7 & -31.0 & -9.0 & -3.6 & -5.3 \\ -10.2 & -31.0 & 99.9 & -17.5 & -12.5 & -25.1 \\ -1.5 & -9.0 & -17.5 & 54.3 & -11.6 & -14.2 \\ -0.5 & -3.6 & -12.5 & -11.6 & 28.4 & 0 \\ -1.0 & -5.3 & -25.1 & -14.2 & 0 & 45.8 \end{bmatrix}$$

This is another illustration of similar and more detailed examples of this concept in Refs. 1 and 6. Note that the modified stiffness matrix contains a small negative spring between DOF 5 and 6, whereas the original value was zero. One can understand the zero value because all load paths between these two DOF are blocked by DOF 3. This is true in the representation of the simple structure (Fig. 1), as well as the model (Fig. 2). However, it is also true that the zero value is incorrect because there are other load paths in this continuous structure, which are not in either of the representations. Going back to a more detailed representation of the structure would correct this problem.

Either of the models using the preceding stiffness matrices, or any based on the infinite reasonable combinations of acceptable frequencies, could be expected to satisfactorily represent the dynamic behavior of the structure.

Conclusions

It is apparent that any linear finite element representation of a structure is valid only over a limited range.

Any finite element representation can be improved by going back to a more detailed model and performing a dynamic reduction in DOF. In this case, however, the mass matrix will be filled and will vary with frequency, and the stiffness matrix will also basically be filled. An adequate representation of the behavior over a limited range is possible using a constant stiffness and mass matrix. It is to be expected that there will be an infinite number of reasonable models that will adequately predict this behavior.

It follows that the modification of a particular FEM to make its predictions better agree with measured data can have many physically reasonable results.

There is certainly no doubt that the dynamic modeling of structures is very important. The ability to improve and correct the capabilities is also very important. The comments in the following paragraphs have rarely been addressed and must be answered before any practical applications can be made of techniques that use actual test data to improve the dynamic representation of structures.

A major question is, of the various methods related to the important problem of improving a model, how should one determine which one to use? This is a question that has not been adequately addressed by persons working in this field.

Certainly, a partial answer must relate to how the modified model is to be used. If all that one wished to do was to see whether a small variation in the parameters of the model would make it agree with the test and, thus, verify the physical representation of the structure, almost any valid procedure would be satisfactory.

There are a number of applications for which the answer is not so simple and the answers may vary for the different applications. Some of these are the ability to predict: effects of design changes, effects of major damage, behavior of the structure as a component of a larger system or different boundary conditions, behavior of the structure under various loading conditions, and behavior of the system in response to automatic control systems.

Another even more difficult and potentially very important application is the ability to determine the damage to a structure based on changes in its behavior. The applications discussed earlier only require that the behavior be predicted and, while it would be desirable, the updated model can be less than truly representative of the structure's physical characteristics. To meaningfully identify damage, the procedure must identify the true changes in the physical characteristics of the structure, or at least identify the likely location of these changes. This is a considerably more difficult requirement than the aforementioned requirement.

The preceding questions must be answered before any practical application can be made of this important area of dynamic structural engineering. It is strongly recommended that meaningful research be initiated in this area.

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